Dynamic Programming Technique for Manufacturing Systems

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***Abstract*—Dynamic programming is a powerful technique that can be used in manufacturing systems. In modern manufacturing systems it is critical to perform maintenance, management and calibration problems where objective is to find best solution of the problems. Dynamic programming divides the problem into sub problems and finds the optimum solution of the problem and their result can be reused. it reduces the computational time. In this paper we propose dynamic programming approach, characterization, classification, mathematical formulation and its applications to manufacturing purposes. Some researches are sprinkled through the paper illustrating the specific type of problems and also discuss how dynamic programming is applied to the manufacturing system. The major contribution of this paper includes a simple and powerful abstraction of dynamic programming applications and implementation. Our paper results show the basic idea of dynamic programming and how it is important and how to approach it and the results for application demonstrates the applicability of algorithm to variety of optimization problems.**

***Keywords***—**dynamic programming, manufacturing system, optimum problem**

I. INTRODUCTION

Dynamic programming is a mathematical technique dealing the optimization of multistage decision problems. The technique was originated in 1952 by Richard bellman and G.B dantzig and was initially referred as stochasting dynamic programming [1].We does not have a standard formulation of the dynamic programming problem. For each problem, depending on the variables given, and objective of the problem, one has to develop a particular equation to fit for a situation. Though we have quite good number of dynamic programming problems, some times to take advantage of dynamic programming, we introduce multistage nature in problem and solve it by dynamic programming technique. Now a day’s application of dynamic programming is done in almost all day to day managerial problems, such as inventory problem, waiting line problems, resource allocation etc.[2]. the difficulty of dynamic programming in the development of the proper model to represent a particular situation like an artist experience in model development is essential to be able to manage complex problems and establish recurrence relation between interrelated problems in consecutive stages for this reason, this chapter introduce dynamic programming by analyzing different applications where decision variables may be integer or continues objective function and constraints may be either linear or nonlinear and data may be deterministic or random variables with known probability distributed function. The solution of dynamic programming problems depends upon principal of optimality.

### Richard bellman’s principal of optimality said that -An optimal policy has the property that whatever the initial state and the initial decisions are; the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decisions [6].

Terminology used in dynamic programming as follows [1]:-

**State**: the DP problem is broken into sub problems is called stages.

**Stage decision**: at each stage there are number of alternatives and the selection of one of the most suitable and feasible alternative is called stage decision.

**State variables**: the variables whose value specify the condition of decision process and summarize the current status of the system called state variables.

**Decision variables**: the unknown in the given problem that needed to be determined are called decision variables.

Elements of dynamic programming

**Simple sub problems**- We must able to break the original problems to smaller sub problems that have the same configuration.

### **Optimal substructure** - If the optimal solution has contained optimal sub solutions then that problem exhibit optimal substructure.

### **Overlapping sub problems** - When a recursive algorithm would gain the same sub problems again and again, then a problem has overlapping sub problems.

Forward and backward computational procedure in dynamic programming [3].

When the dynamic programming problem solved using the recursive equation starting from the 1st stage to the last stage the computation involved is called the forward computational procedure.

When the dynamic programming problem solved using recursive equation starting from the last stage to the first stage the computation involved is called the backward computational procedure.

Procedure adopted in dynamic programming problems [2].

Define the variables, objective function and constraints.

Split the problem into number of sub problems.

Develop recursive relationship for optimality.

Decide whether to follow the forward or the backward method to solve the problem.

Make tabular presentation to show the required values and calculations for each stage.

Find optimal policy at each stage and then the overall optimal policy.

II. CHARACTERIZATION

1. Dynamic programming divides the original large problem into smaller sub problems called stages involving only a few variables, wherein outcomes of decision at one stages affects the decisions at the remaining stages.
2. Every stage consists of a number of states associated with it .The states are the different possible conditions in which the system may find itself at that stage of the problems.
3. Effect of policy decision at each stage converts the current stage into state associated with the beginning of the next stage.
4. A stage decision does not alter the number of variables on which the outcomes depends, but only changes the numerical value of these variables.
5. The state of system at a stage is described by a set of variables, called state variables.
6. When the current state is known, an optimal policy for the remaining stages is independent of the policy decisions adopted in the previous ones.
7. To identify the optimum strategy for each state of the system, a recursive equation is formulated with the ‘N’ stages remaining, given the optimal policy for each stage with (N-1) stages left.
8. Using recursive equation approach each time the solution procedure moves backward, stage by stage using for obtaining the optimum policy of each stage for that particular stage, still it attains the optimum policy beginning at the early stage [5].

III. CLASSIFICATION

**Deterministic dynamic programming:** It is the dynamic programming approach to deterministic problems, where the state at the next stage is completely determined by the state and policy decision at the current stage [4]. Deterministic dynamic programming can be described diagrammatically.at stage n the process will be in some state. Making policy decision andthen moves the process to some state at stage n+1. The contribution thereafter to the objective function under an optimum policy has been previously calculated to be .the policy decision also Make some contribution to the objective function. Combining these two quantities in an appropriate way provides .the contribution of stages n onward to the objective function. Optimizing with respect to then gives .after and are found for each possible value of  the solution procedure is ready to move back one stage.

Stage

Stage

State

Contribution of

Value:

Value:

Fig.1 The basic structure of deterministic dynamic programming

**Probabilistic dynamic programming:-**Probabilistic or stochastic dynamic programming different from deterministic dynamic programming in that the state at the next stage is not completely determined by the state and policy decision at the current stage[4]. Relatively, there is a probability distribution for what the next state will be. Yet this probability distribution still is completely determined by the state and policy decision at the current stage. S denote the number of possible states at stage n+1 and label these states on right side as 1, 2,…..S. the system goes to state i with a probability given state sn and decision at stage n. if the system goes to state is the contribution of the stage n to the stage n to the objective function. When fig. is expanded to include all possible states and decision tree. If the decision tree is not too large .it provides a useful way of summarizing the various possibilities.

Because of the probabilistic structure, the relationship between and the. necessarily is somewhat more complicated than that for deterministic dynamic programming .the precise form of this relationship will depend upon the form of the overall objective function.to illustrate, suppose that the objective is to minimized the expected sum of the contributions from the individual stages. In this case represents the minimum expected sum from stage “n” forward, given that the state and policy decision at stage n are and , respectively.

Stage n+1

P2

C2

Stage n

State:

Decision

)

)

F\*n+1(1)

F\*n+1(2)

F\*n+1(s)

-probability

- probability

-contribution from stage n

,

,C

P1

C1

PS

CS

Fig.2 The basic structure of probabilistic dynamic programming

=

With

=

Where this minimization s taken over the feasible values of xn+1.

IV. FORMULATION

From [6] we formulate the dynamic programming, Let R be the resource such as machine, money, material etc., n be the number of different activities to which resource is to be distributed,

And P is the return, which depends on the activities, quality of resource allocated and objective.

If denote the return from the activity with resource, then the total return may be expressed as,

Constraints, and

Objective is to maximize the total return,

Allocate the resource to the activities, one by one, starting from the last i.e. nth activity.

Let be the quantity of resource allocated to nth activity.

is the return from nth activity.

be the quantity of resource can be allocated to the remaining activities i.e. (n-1).

is the return from n-1 activities.

Total return from all n activities is

So dynamic programming model can be return as

V. APPLICATION

Dynamic programming is a powerful technique that can be used to solve many problems in time for which a naïve approach would take exponential time .following are a few number of fields in which dynamic programming has successfully applied[2][12].

1. The application of dynamic programming technique has been used in production system such as for determining optimal just in time production schedule for a mixed model facility system, Deadlock-free scheduling of manufacturing system using petri nets and dynamic programming or to achieving an optimal end state along a serial production line.
2. Dynamic programming technique has been used to determine the optimum inventory level and for formulating the inventory recording rules, indicating when to replenish an item and by what amount.
3. Selection of an adverting media such as to determining the optimal combination of advertising the media and advertising frequency.
4. Dynamic programming technique is utilized to guarantee high efficiency utilization of expensive equipment.
5. Dynamic programming technique is used to determine at which stage equipment is to be replaced for optimum return from the facilities.
6. Application of dynamic programming in knapsack/ fly-away/ cargo-loading model. The knapsack model classically deals with the situation in which a soldier (or a hiker) must decide on the most valuable items to carry in a backpack. Knapsack problem is also called in the literature as the fly-away kit problem in which jet pilot must determine the most valuable cargo items.

VI. SOME LATEST RESEARCHES

1. A dynamic programming approach for storage location assignment planning problem: in [7] the author described it is usually benefit for online retailer to provide fast product delivery service which ensures that customer can receive ordered product in a short time the product delivery service improvement requires warehouses or distribution centers to fulfill customer orders within tight time windows, in which order picking is the most time consuming and labor intensive activity. Various storage policies are proposed to improve the order picking efficiency but they disregard the fact that many products, demand fluctuate over time. Moreover in a long planning horizon, warehouse manager may concern that when to update the warehouse storage location assignment to meet the demand trend. Author refers to the problem as a storage location assignment planning problem and formulates storage location assignment planning problem as a mathematical programming and develop dynamic programming approach to solve it.

2. A dynamic programming model for designing a quality control plan in manufacturing systems: From [8] we conclude that modern manufacturing systems require significance large number of inspection allocation problem has been studied using several analytic optimization and simulation methods although a no. of methods are available to address this problem, there is still a considerable gap between theoretical methods and their practical application. The focus of this work is related to reducing this gap because the author consider obtaining a good quality control plan for multistage manufacturing systems is a key elements in a achieving the desired quality level with the lowest control cost. The proposed approach makes it possible to obtain quality control plan of a multistage manufacturing systems using stochastic dynamic programming

3. Adaptive railway traffic control using approximate dynamic programming: in [9] we conclude that approximate dynamic programming technique to solve large scale discrete time multistage stochastic control process.in this research article author used approximate dynamic programming technique which presents an investigation into an adaptive railway traffic controller for real time operations that applies approximate dynamic programming. By assessing requires and opportunities the controller aim to limit train delays by advantageously controlling the sequencing of trains at critical locations in a timely manner few in the literature have investigated method ds that are appropriate for use in stochastic environments. Many suffer from high computational complexity which makes them inefficient or difficult to adopt for practical operation. The author therefore has developed an approximate dynamic programming framework to reduce the computational burden which in turn confers flexibility in control in a stochastic environment that has been tested using a monte-carlo scheme.

4. An application of dynamic programming to assign pressing tanks at wineries: In this paper author defined an application of dynamic programming to determine optimal strategy for assigning grapes to pressing tanks in one of the largest Portuguese wineries based on earlier researches it is concluded that the topic of real -time scheduling in the wine industry was not deeply explored in the fact to date the most popular strategy for solving win -connected scheduling problems, however the problem we seek to address requires to understand series of decisions at real time and so linear programming solution is not appropriate consequently in this article author a new model that include the stochasticity and also created an algorithm based on dynamic programming employing the bellman equation[11].

5. An analytical bound on the fleet size in vehicle routing problems: A dynamic programming approach: in this article author studied the maximum numbers of homogenous vehicles in node routine problems with integer splits we assumed that all parameter as well as the vehicle loads are integer .furthermore, each vehicle can make a delivery to exactly one depot, and the aggregate load for one vehicle author first obtained a closed form for the optimal objective value of maximization problem. This gives a valid upper bound based on our assumptions where there is only predetermined depot the author then used this upper bound to develop an optimization problem whose optimal value gives a tight upper bound to develop an optimization problem whose optimal value gives a tight upper bound for multiple capacitated depots the desirable characteristic of the proposed upper bound is that it is optimal in the sense that it gives a tight bound based on our assumption for any given instance[10].

VII. Conclusion

The paper has been to introduce the reader about dynamic programming that is the approach to solve optimization problems and very useful technique for making a sequence of interrelated decisions. Our motive is to find the best solution from a set of alternatives.one difficulties in dynamic programming is in the development of the mathematical formulation and the establishment of the recurrence relation that allow us to solve instance of the problem in stages in an efficient manner for this reason we have taken the approach of introducing about dynamic programming and discussing the characterization, classification and formulation and its application. Characterization shows the differentiation of dynamic programming from other quantitative techniques of decision making, Classification have been categorized in deterministic models where data is known with certainty and probabilistic models in which some of the information is uncertain and require the use of probability distribution function. And the mathematical formulation used to solve the optimization problems, applications shows the dynamic programming used in different types of optimization problems.

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