**Variable Viscosity and Prandtl Number Effects on Natural**

**Convection Methanol Boundary Layers about a Vertical Plate with Suction**

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***Abstract--*This study deals with the effect of temperature-dependent viscosity and Prandtl number on the steady, laminar flow of methanol, past a vertical porous plate. The coupled nonlinear partial differential equations governing the non-similar flow have been solved numerically using an implicit finite difference scheme in combination with the quasilinearization technique. Numerical results indicate that variable viscosity and Prandtl number, both have a major role on skin friction and heat transfer parameters as well as velocity and temperature fields. Further, it is observed that the effect of variable fluid properties along with suction plays a significant role in the control of laminar boundary layer. The present analysis reveals the fact that when the working fluid is sensitive to the temperature, the effect of variable viscosity and Prandtl number has to be taken into the consideration in order to predict the skin friction and heat transfer rate accurately.**

***Index Terms- Heat transfer, Skin friction, Temperature-dependent Viscosity, Temperature, Velocity***

*MSC 2010 Codes* *– 76M20, 76N20, 76R10*

# **INTRODUCTION**

Applications of heat transfer are generally based on the constant physical properties of the ambient fluid in fluid dynamics research. However, it is known that these properties may change with temperature, especially the fluid viscosity and hence, the Prandtl number. Numerous researchers have studied the effect of variable viscosity on different geometries under various situations [1-7].

Free convection boundary layer flows frequently encountered in environmental and engineering devices. Abundant literature is available on the topic of the laminar boundary layer flow over a porous vertical plate with suction and injection, having wide range of engineering applications. In fact, the case of uniform suction and blowing (injection) through an isothermal vertical wall was treated first by Sparrow and Cess [8]; they obtained a series solution which is valid near the leading edge. This problem was considered in more detail by Merkin [9], who obtained asymptotic solutions, valid at large distances from the leading edge, for both suction and blowing (injection).The present study is undertaken to investigate the effect of variable viscosity and Prandtl number on the free convection boundary layer flow (of methanol) over a vertical porous plate with suction. It may be remarked here that methanol is a liquid in room temperature, used in

thousands of everyday products, including plastics, paints, cosmetics and fuel industries.

# **GOVERNING EQUATIONS**

We consider a semi-infinite porous plate, which is played vertical in a quiescent fluid (methanol*)* of infinite extent maintained at an uniform temperature. The plate is fixed in a vertical position with leading edge horizontal. The physical co-ordinates (*x,y*) are chosen such that *x* is measured from the leading edge (origin) in the stream wise direction and *y* is measured normal to the surface of the plate.Indeed, the flow is assumed to be in the *x*-direction i.e., along the vertical plate in the upward direction and the *y*-axis is taken to be normal to the plate.

The fluid properties are assumed to be isotropic and constant except for the fluid viscosity. The temperature difference between the surface of the plate (Tw) and the ambient fluid (T∞) is taken to be small. In the range of temperature (T) considered (i.e. 0-600C),the variation of both density () and specific heat (*cp)* of methanol with temperature is small and hence they are taken as constants*.[See Table I]* However, the viscosity () and thermal conductivity (k) [and hence the Prandtl number (Pr)]are assumed to vary as an inverse linear function of temperature:

 (1)

 (2)

where

 (3)

***Table I***

***Values of thermo-physical properties of methanol at different temperature* [10]**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Temperature****(T0C)** | **Density(*ρ*)****(gr./cm3)** | **Specific****heat(*cp*)****(J × 107/kg 0K )** | **Thermal****conductivity (*k*)****(erg × 105/cm.s-0K )** |  **Viscosity(*μ*)** **(gr. × 10-2/ cm-s)** | **Prandtl** **number****(Pr)** |
| 0 | 0.813 | 2.399 | 0.207 | 0.777 | 9.005 |
| 10 | 0.804 | 2.449 | 0.204 | 0.664 | 7.971 |
| 20 | 0.794 | 2.504 | 0.201 | 0.575 | 7.163 |
| 30 | 0.785 | 2.566 | 0.199 | 0.504 |  6.498 |
| 40 | 0.775 | 2.633 | 0.196 | 0.447 | 6.004 |
| 50 | 0.765 | 2.706 | 0.193 | 0.399 | 5.594 |
| 60 | 0.755 | 2.785 | 0.190 | 0.360 | 5.276 |

The relation (1) and (2) are reasonably holds good approximations for liquids such as methanol, particularly for small wall and ambient temperature differences. Further, the fluid added (injection) or removed (suction) is the same as that involved in flow. The Boussinesq’s approximation employed for the fluid properties to relate density changes in the flow field. Under the above-mentioned assumptions, the boundary layer equations governing the steady, two-dimensional flow are [9]:

 (4)

 (5)

 (6)

The initial and boundary conditions are

 (7)

Introducing the following transformations

 (8)

to Equations (4) – (6), we see that the continuity Equations. (4) is identically satisfied and Equations. (5)–(6) reduces, respectively, to

 (9)

(10)

where

 (11)

It is noted here that the upper and lower signs in Eqns. (9) and (10) is taken thought for suction and injection, respectively. The present study, however, restricted to the case of suction only.

The transformed boundary conditions are:

 (12)

The local skin friction and heat transfer parameters can be expressed, respectively, as

 (13)

 (14)

Here*, u* and *v* are velocity components in *x* and *y*-directions respectively; F is dimensionless velocity; T and G are dimensional and dimensionless temperatures, respectively; ξ and η are transformed co-ordinates; ψ and *f* are the dimensional and dimensionless stream functions respectively; Pr is the Prandtl number; are constants; *g* is the gravitational acceleration; is the coefficient of thermal expansion; *w* and ∞ denote conditions at the edge of the boundary layer on the wall and in the free stream respectively, the subscript  and prime denote, respectively partial derivatives with respect  and .

# **METHOD OF SOLUTION**

The system of dimensionless nonlinear coupled partial differential equations (9)-(10) with boundary condition (12) has been solved numerically employing an implicit finite difference scheme with a quasilinearization technique. Applying this technique, the coupled nonlinear partial differential equations are reduced to the following linear partial differential equations:

 (15)

 (16)

The coefﬁcient functions with iterative index *k* are known and the functions with iterative index *(k + 1)* are to be determined. The boundary conditions are given by

 (17)

The coefﬁcients in Equations. (15)– (16) are given by

 ;

;

 ;

 ;

;

;

;

where;

Since the method is presented in a classical paper by Inouye and Tate [12] and adopted by numerous researchers, its detailed description is not provided here. At each iteration step, the sequence of linear partial differential equations. (15) and (16) were expressed in difference from using central difference scheme in the -direction and backward difference scheme in ξ- direction. Thus, in each step, the resulting equations were then reduced to a system of linear algebraic equations with a block tridiagonal matrix, which is solved by Varga’s algorithm [13]. To ensure the convergence of the numerical solution to the exact solution, step sizes Δ and Δξ are optimized and taken as 0.001 and 0.01, respectively. The results presented here are independent of the step sizes at least up to the fourth decimal place. A convergence criterion based on the relative difference between the current and previous iteration values is employed. The solution is assumed to have converged and the iteration process is terminated when the difference reaches i.e.,

 (18)

# **RESULTS AND DISCUSSION**

In order to assess the accuracy of the numerical method which we have used, the skin friction and heat transfer parameters  for suction have been obtained by solving equations (9) and (10) for constant viscosity [N=1] case, taking Pr=1.0, and compared with those of Merkin [9]. Our results are found to be in good agreement with those of [9], as shown in Fig.1, validating the accuracy of the numerical method used in the present study. The computed results for variable viscosity as well as Prandtl number have been presented in the graphical form and analyzed.



. **Fig. 1. Comparison of skin friction and heat transfer**

 **parameter with Merkin [9] for suction**

 **Fig.2. Variation of (a) skin friction and (b) heat transfer**

 **parameters along stream-wise directions**

Figure 2 describes the variation of skin friction and heat transfer parameters with the stream wise coordinate , in the presence of both variable fluid properties [T∞ =28.0oC, Tw= 10.0] and constant fluid properties [N =1 and Pr = 7.2 for methanol at room

temperature], and suction. It is observed from Fig.1(a) that skin friction increases from zero to a maximum value in a certain range of =1.0, and then decreases as further increases. It is also observed that the effect of variable fluid

properties is to increase the skin friction and to decrease the heat transfer. In fact, for variable fluid properties differs from that of constant fluid properties by about 14.3% [Fig.2(a)] while, the percentage of difference in the

case of is about 1.822% [Fig.2(b)], at the stream-wise coordinate ξ =0.6. Further, it is observed that the zero-skin friction is moved downstream in the presence of variable fluid properties. Indeed, in the case of constant fluid

properties zero skin friction occurs at the stream-wise location ξ = 1.8 whereas for variable fluid properties, the same occurs at ξ = 1.9. This justifies the delay in the boundary layer separation under the influence of variable

viscosity,Prandtl number and suction.

 **Fig. 3. Behavior of (a) velocity and (b) temperature**

 **profiles at different stream-wise locations**

The relevant velocity (*F)* and temperature(*G*) profiles are shown in Fig.3, for the case of variable fluid properties.

 It is observed that the thickness of momentum boundary layer decreases with the increase of stream wise coordinate () [Fig.3(a)], which results in the increase of velocity of the fluid inside the boundary layer. On the other hand, the thermal boundary layer thickness decreases as increases, enhancing the temperature inside the boundary layer [Fig.3(b)].





 **Fig. 4. Effect of ΔTw on (a) skin friction and (b) heat**

 **transfer parameters at stream-wise locations**

The variation of viscosity and Prandtl number with temperature can be introduced in terms of the difference (ΔTw) in the temperature of the wall and ambient fluid[Fig.4]. Since T∞ =28.00C, the maximum value of ΔTw is taken as 200C so that numerical computations are done with in the permissible temperature. In Fig 4(a)–(b) for different stream wise locations it is observed that as ΔTw increases τ w also increases, however Q decreases up to ΔTw = 50C and again it increases as ΔTw increases. Further, it is observed that as ξ increases both skin friction and heat transfer decrease, the rate of decrease of skin friction is 2.3% and 2.6% respectively at ΔTw = 100C and 150C, while the rate of decrease in heat transfer is 23.25% and 25.2% for the same values of ΔTw.

# **CONCLUSIONS**

The steady, laminar methanol boundary layer flow (of methanol) past a vertical porous plate is numerically investigated assuming both viscosity and Prandtl number as linear inverse functions of temperature. The computed results show that the flow/temperature fields, skin friction and heat transfer characteristics are significantly affected by the temperature-dependent viscosity and Prandtl number in the presence of suction. From the present study, it is concluded that the effect of variable fluid properties along with suction plays a significant role in the control of laminar boundary layer over a vertical porous flat plate

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